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Non-singular solutions to Einstein-Klein-Gordon equations with a phantom scalar field

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Abstract: It is shown that the 4D Einstein-Klein-Gordon equations with a phantom scalar field (a scalar field with a negative sign in front of the kinetic energy term of its Lagrange density) has non-singular, spherically symmetry solutions. These solutions have a combination of features found in other spherically symmetric gravity plus field solutions. A stability analysis on these solutions indicates they are unstable.

Keywords: Cosmology of Theories beyond the SM, Classical Theories of Gravity.

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Contents

1. Introduction

When one considers the system of gravity plus some field(s) (e.g. scalar field, gauge fields) there are a range of spherically symmetric solutions. When these extra fields are non-interacting, complex, scalar fields one finds the boson star solutions [1 − 3] which are prevented from collapsing because of the Heisenberg uncertainty principle. Allowing the complex scalar field to have a self interaction of the $\lambda\phi^4$ type lead to boson stars with masses comparable to the Chandrasekhar mass [4, 5] in contrast to the much smaller mass of the non-interacting boson star solutions of [1 − 3].

If one considers a real scalar field rather than a complex scalar then there are no non-singular, static solutions [6 − 9]. However it was also shown in [6] that if one considers ghost fields (a real scalar field with a negative sign for the kinetic *and* potential parts of the scalar field Lagrangian density) one has non-singular, static solutions, but having a non-trivial topology. These ghost field solutions have a wormhole structure. This might have been expected since ghost fields may have a negative mass-energy density, and it is just such a form of matter that is needed in order to support wormholes [11]. A final related system occurs with a phantom Born-Infeld field forming compact objects ('gravstar') [12].

In this work we investigate spherically symmetric solutions to gravity plus a real scalar field. In contrast to previous work we consider a phantom field [13] i.e. a real scalar field with a negative sign in front of *only* the kinetic energy part of the Lagrangian density. Phantom fields were originally proposed as an extreme form of dark energy which could explain the accelerated expansion of the Universe [14]. Phantom fields can violate the weak energy condition (WEC) and appear to be quantum mechanically unstable. (The WEC requires that the mass-energy density, ε, and pressure, p, of the field/fluid satisfy $\varepsilon + p \geq 0$ or $w \equiv p/\varepsilon \geq -1$). Any field or fluid with $w < -1/3$ will lead to gravitationally repulsion and is called dark energy. Phantom energy with $w < -1$ is an extreme form of dark energy. A field or fluid with $w > -1/3$ leads to gravitational attraction. Nevertheless, as pointed

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out in [13] the supernova data seem to favor an extreme form of dark energy like phantom energy. The original proposal for phantom energy can be found in [15, 16] (see also [17] and references therein for related work in this direction). There are also the so-called 'quintom' models of dark energy [18] with an equation of state that crosses the cosmological constant boundary and, correspondingly, violates the WEC.

Phantom energy can lead to gravitational repulsion and therefore accelerated expansion of the Universe. This repulsion can provide a mechanism for the existence of static, regular solutions — the gravitational repulsion of the phantom field can be balanced by the gravitational attraction of normal matter. This is the physical mechanism behind the Bartnik-McKinnion solution [19] to the Einstein-Yang-Mills system – the repulsion coming for the self-interaction of the Yang-Mills fields balances the usual gravitational attraction and stabilizes the configuration. As with the Bartnik-McKinnion solution the phantom field solution presented here is regular everywhere and asymptotically goes to Minkowski spacetime. Unlike the ghost field, kink solution of [6], with its non-trivial, wormhole topology, the present phantom field solution (like the Bartnik-McKinnion solution) represents a spherically symmetric, localized field configuration, and has a trivial topology.

One unusual feature of the phantom field solutions examined here is that it has an overall negative mass. This is in contrast to the ghost field solution of [6] with its positive mass. Both phantom fields and ghost fields face difficulties when one tries to quantize them. Nevertheless the data on the accelerated expansion rate of the Universe favors something like phantom energy [13] and so one should study the consequences of such unusual form of matter-energy. As far as we know phantom energy has been studied in a cosmological context where the phantom field varies only with time but does not vary spatially. In the present paper our phantom field varies spatially. Also strictly speaking our scalar field is only a phantom field (i.e. a field with $w < -1$) in some spatial regions, while in other regions it behaves as a gravitationally attractive field with $w > -1/3$. Physically this makes sense since a field which is phantom everywhere in space could not form some static spherically symmetric configuration but would disperse.

There have been other works which have studied spherically symmetric solutions with phantom or ghost fields [20 − 22]. In [23] black hole solutions in the presence of a phantom field were investigated. In [24] the process of accretion of phantom energy onto a black hole was considered. In [25] a study was undertaken of spherically symmetric solutions to the system of gravity plus some matter/fluid which violated weak energy condition. In this paper we replace the fluid by a scalar field. In regard to the negative mass of our solution there are other known negative mass solutions such as the negative mass black holes of [26]. However as with the solutions ghost field kink solutions of [6] these negative mass black hole solutions have non-trivial topology.

2. Equations and solutions

We chose our spherically symmetric metric to have the form [6]

$$ds^2 = e^{\nu(t,r)}dt^2 - e^{\lambda(t,r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.1}$$

For the matter Lagrangian we have one real phantom scalar field with the Lagrangian

$$L = -\frac{R}{16\pi G} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\ . \tag{2.2}$$

For the potential we take

$$V(\varphi) = -\frac{1}{2}m^2\varphi^2 + \frac{\kappa}{4}\varphi^4, \tag{2.3}$$

where m is the mass of the scalar field and κ is the coupling constant. This is the usual "Mexican hat" symmetry breaking potential. Note that in our terminology phantom field means a negative sign in front of the kinetic energy term only. By ghost field we mean a negative sign in front of both the kinetic and potential terms of the scalar field Lagrangian. With this definition it is phantom fields (not ghost fields) which can lead to $w < -1$ and accelerated expansion of the Universe. In what follows we set $16\pi G = 1$.

The energy-momentum tensor follows from (2.2)

$$T_{\mu\nu} = -\partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\left[-\frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi - V(\varphi)\right]. \tag{2.4}$$

The corresponding Einstein equations are

$$R_{\mu\nu} = \frac{1}{2}\left[-\partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}V(\varphi)\right] \tag{2.5}$$

Note we are using the conventions of [27] where the Ricci tensor is defined via the contraction of the first and third index of the Riemann tensor $R_{\mu\nu} \equiv R^\alpha_{\mu\alpha\nu}$. This is the reason for the overall + sign on the right hand side of (2.5) (see also [28]). From (2.5) it follows

$$\begin{aligned} R_{01} &= \frac{1}{r}\dot\lambda = -\frac{1}{2}\dot\varphi\varphi', \\ e^{-\nu}R_{00} + e^{-\lambda}R_{11} &= \frac{1}{r}e^{-\lambda}\left(\nu' + \lambda'\right) = -\frac{1}{2}e^{-\nu}\dot\varphi^2 - \frac{1}{2}e^{-\lambda}\varphi'^2, \\ R_{22} &= -\frac{1}{2}re^{-\lambda}\left(\nu' - \lambda'\right) + 1 - e^{-\lambda} = \frac{1}{2}r^2V(\varphi). \end{aligned} \tag{2.6}$$

The wave equation for the scalar field φ follows from variation of the Lagrangian (2.2)

$$e^{-\nu}\ddot\varphi - \frac{1}{2}e^{-\nu}\left(\dot\nu - \dot\lambda\right)\dot\varphi - e^{-\lambda}\varphi'' - e^{-\lambda}\left[\frac{2}{r} + \frac{1}{2}\left(\nu' - \lambda'\right)\right]\varphi' - \frac{dV}{d\varphi} = 0. \tag{2.7}$$

In the above the ansatz functions are t and r dependent. Differentiation with respect to t (r) is denoted by a *dot* (*prime*).

To investigate the static problem we drop the time dependence and express the metric functions ν, λ via new metric functions $C(r), M(r)$

$$e^\nu = C(r)\left[1 - \frac{2M(r)}{r}\right], \qquad e^\lambda = \left[1 - \frac{2M(r)}{r}\right]^{-1}. \tag{2.8}$$

(In the next section when we investigate the stability of our solutions we will again consider time dependent ansatz functions). In order to perform the numerical analysis we also introduce new dimensionless variables

$$x = mr, \quad \tilde M = 2mM, \quad \Lambda = \frac{\kappa}{m^2}.$$

Using these functions and variables the Einstein equations (2.6) become

$$\frac{1}{x}\frac{C'}{C}\left(1-\frac{\tilde{M}}{x}\right)-\frac{\tilde{M}'}{x^2}=\frac{1}{4}\left[-\left(1-\frac{\tilde{M}}{x}\right)\varphi'^2+\varphi^2-\frac{\Lambda}{2}\varphi^4\right] \tag{2.9}$$

and the equation for $\tilde{M}$ becomes

$$\tilde{M}'=-\frac{1}{4}x^2\left[\left(1-\frac{\tilde{M}}{x}\right)\varphi'^2+\varphi^2-\frac{\Lambda}{2}\varphi^4\right]\,. \tag{2.10}$$

Combining (2.9) and (2.10) we find

$$\frac{C'}{C}=-\frac{1}{2}x\varphi'^2. \tag{2.11}$$

Also the scalar field equation (2.7) takes the form

$$\varphi''+\left\{\frac{2}{x}\left[1-\frac{1}{2}\left(\frac{\tilde{M}'-\tilde{M}/x}{1-\tilde{M}/x}\right)\right]-\frac{1}{4}x\varphi'^2\right\}\varphi'=\left(1-\frac{\tilde{M}}{x}\right)^{-1}\varphi\left(1-\Lambda\varphi^2\right). \tag{2.12}$$

Thus we only needed to solve for $M(x), \varphi(x)$ via (2.10) (2.12) since from (2.11) $C(x)$ was determined by $\varphi(x)$.

We solved equations (2.10) and (2.12) numerically using the following boundary conditions:

$$\varphi(0)=const, \qquad \varphi'(0)=0, \qquad \tilde{M}(0)=0. \tag{2.13}$$

That is we demand regularity at the origin. Next choosing some Λ we found finite energy, non-singular solutions by numerically solving the system of equations (2.10) and (2.12). For a given Λ this amounted to finding the correct initial value of the scalar field $\varphi(0)$ which gave the desired well behaved solution. The problem reduces effectively to an eigenvalue problem since for a given Λ one had to search for the proper "eigenvalue" $\varphi(0)$ in order to get a well behaved solution (regular, asymptotically flat) for the metric and scalar field ansatz functions. The flatness of the spacetime at infinity was provided by choosing the proper initial value for $C(0)$ in equation (2.11) i.e. we could chose $C(0)$ such that $C(\infty)\to 1$.

In figure 1 we show the graphs of the scalar field for two values of Λ. Generically the solutions for the scalar field looked similar — starting at some non-zero value at $x=0$ and decaying to zero as $x\to\infty$ i.e. $\varphi(\infty)=0$. For a normal scalar field this asymptotic value would be strange since for the "Mexican hat" potential this is a local maximum. Keeping in mind the unusual character of the phantom field it is not unexpected that the field should asymptotically go to a maximum rather than a minimum. The behavior of the metric functions $e^{\nu(x)}$ and $e^{\lambda(x)}$ for these values of Λ is shown in figure 2. Again the behavior of the metric functions is generally similar for different values of Λ – starting at some constant value at $x=0$ they asymptotically go to 1 without ever becoming 0 or negative. Thus the metric is non-singular and without horizons.

The mass of the solutions was obtained from the asymptotic values of $\tilde{M}(\infty)$ from (2.10) . The mass scale of the dimensionless quantity $\tilde{M}(\infty)$ is set by the mass

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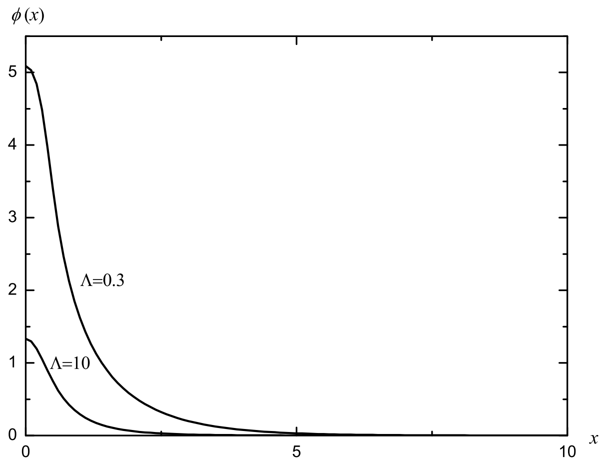

Figure 1: The scalar field φ for two different values of Λ.

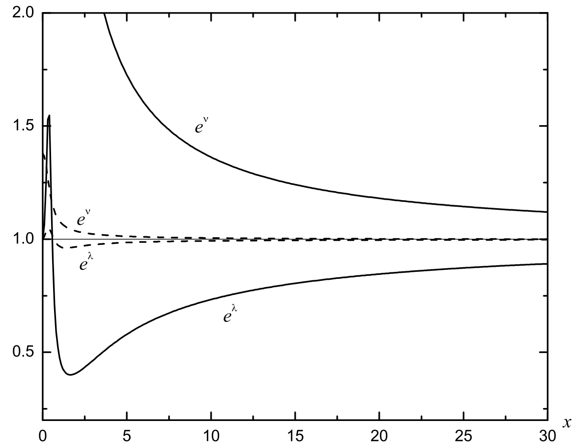

Figure 2: The metric functions $e^{\nu(x)}$ and $e^{\lambda(x)}$ when $\Lambda = 0.3$ (solid line) and $\Lambda = 10$ (dashed line).

m from the potential $V(\varphi)$. The dependence of the $\varphi(0)$ and $\tilde{M}(\infty)$ is shown in table I and also in figure 3. From the figure and table one can see that the total mass goes to zero as $\Lambda \to \infty$. Also $\varphi(0)$ goes to zero as $\Lambda \to \infty$. $\tilde{M}(\infty)$ approaches zero from below while $\varphi(0)$ approaches zero from above.

2.1 Asymptotic behavior

We now examine the asymptotic behavior of the solutions in order to get some analytic

$\varphi(0)$	Λ	$\tilde{M}(\infty)$
7.1312051565	0.1	-23.8587
5.09014118325	0.3	-3.68446
4.35754	0.5	-1.85183
3.478853	1	-0.821492
2.697266	2	-0.391964
2.288345	3	-0.256198
1.3326834	10	-0.0755891
0.6098228	50	-0.0152216
0.4324704	100	-0.00756343
0.1371201	1000	-0.000753043

Table 1: The dependence of $\phi(0)$ and $\tilde{M}(\infty)$ on Λ.

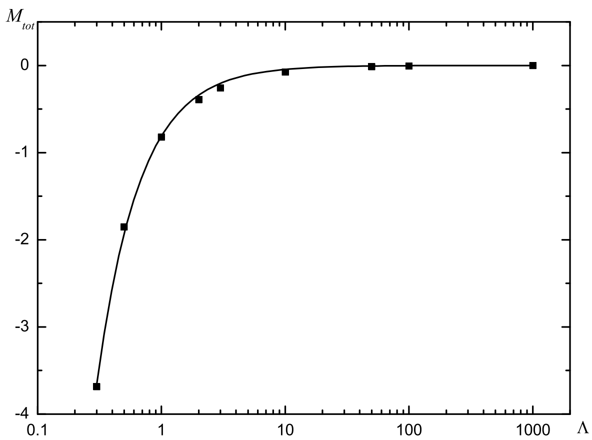

Figure 3: The dependence of dimensionless mass $\tilde{M}(\infty)$ on Λ. Here small squares show calculated values of the mass and the line is an approximation function with equation $\approx const/\Lambda^{1.25}$.

expressions for the solutions for large x and to compare to the numerical solutions. For this purpose, we write the asymptotic forms of the functions as:

$$\varphi \approx \delta\varphi, \quad C \approx C_\infty + \delta C, \quad \tilde{M} \approx \tilde{M}_\infty + \delta\tilde{M}, \tag{2.14}$$

where $\delta\phi, \delta C, \delta\tilde{M} \ll 1$. Then the scalar field equation (2.12) becomes

$$\delta\varphi'' + \frac{2}{x}\delta\varphi' = \delta\varphi. \tag{2.15}$$

We neglect terms of order $(\delta\varphi)^2$, $1/x^2$ and higher. In order to neglect the term $\Lambda(\delta\varphi)^2$ we need to additionally require that Λ is not too large. Although note that as Λ becomes

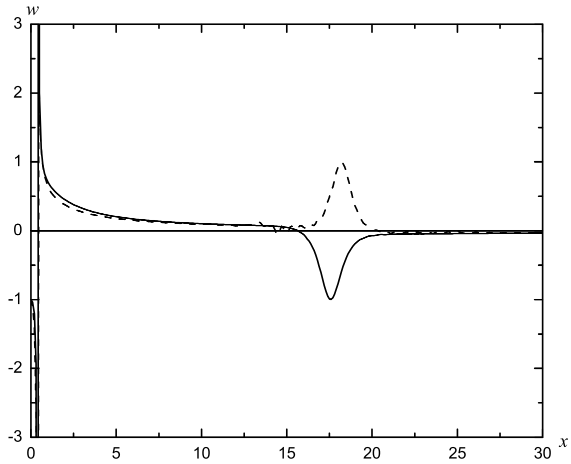

Figure 4: The equation of state $w(x)$ when $\Lambda = 0.3$ (solid line) and $\Lambda = 10$ (dashed line).

large $(\delta\varphi)$ goes more rapidly to zero. This last equation has the asymptotic solution:

$$\delta\varphi \approx C_\varphi \frac{\exp(-x)}{x}. \tag{2.16}$$

Inserting this solution into (2.10) gives the following equation and solution for $\delta\tilde{M}$:

$$\delta\tilde{M}' \approx -\frac{1}{2}C_\varphi^2 \exp(-2x) \quad \Rightarrow \quad \delta\tilde{M} \approx \frac{1}{4}C_\varphi^2 \exp(-2x). \tag{2.17}$$

Using these results in (2.11) we have the following equation and solution for δC:

$$\delta C' \approx -\frac{1}{2}C_\infty C_\varphi^2 \frac{\exp(-2x)}{x} \quad \Rightarrow \quad \delta C \approx -\frac{1}{2}C_\infty C_\varphi^2 \,\mathrm{Ei}(-2x), \tag{2.18}$$

where Ei is the exponential integral function which goes to 0 from below as $x \to \infty$. This shows that all perturbations $(\delta\phi, \delta C, \delta\tilde{M})$ tend asymptotically to zero and that the ansatz functions as given in (2.14) approach the values given by the numerical solution.

2.2 Behavior of the w parameter

We now examine the behavior with respect to x of the effective equation of state $w(x) = p(x)/\varepsilon(x)$ of the scalar field ($\varepsilon(x)$ and $p(x)$ are respectively the energy density and pressure of the scalar field). From (2.1),(2.4) and (2.8) we have:

$$T_0^0 = \varepsilon(x) = -\frac{1}{2}\left(1 - \frac{\tilde{M}}{x}\right)\varphi'^2 + V(\varphi), \tag{2.19}$$

$$T_1^1 = -p(x) = \frac{1}{2}\left(1 - \frac{\tilde{M}}{x}\right)\varphi'^2 + V(\varphi). \tag{2.20}$$

The corresponding effective equation of state for the scalar field is:

$$w(x) = \frac{p(x)}{\varepsilon(x)} = \frac{-\frac{1}{2}\left(1 - \frac{\tilde{M}}{x}\right)\varphi'^2 - V(\varphi)}{-\frac{1}{2}\left(1 - \frac{\tilde{M}}{x}\right)\varphi'^2 + V(\varphi)}. \tag{2.21}$$

Using the numerical solution obtained earlier, figure 4 shows the behavior of $w(x)$ for the case when $\Lambda = 0.3$ (solid line) and $\Lambda = 10$ (dashed line). From the figure one sees that there is some point $x = x_*$ in which the denominator of (2.21) goes to zero (in the case under consideration this occurs around $x_* \approx 0.4$). As $x \to x_*$ from the left $w \to -\infty$ and as $x \to x_*$ from the right $w \to +\infty$. Asymptotically $w(x)$ tends to zero. In the range $0 < x < x_*$ we have $w \leq -1$ ($w = -1$ at $x = 0$) and the WEC is violated. Thus in this region our scalar field acts as phantom dark energy [29]. In the range $x_* < x < \infty$ we have mostly gravitationally attractive matter with $w > -1/3$. For $\Lambda = 0.3$ there is some region around $x \approx 17$ where w briefly dips down and just reaches $w = -1$. In this region the field is dark energy (but not phantom energy) and is gravitationally repulsive. However for both $\Lambda = 0.3$ and $\Lambda = 10$ the field is all or mostly gravitationally attractive. This provides a physical basis for the formation of these solutions: the gravitational repulsion coming from the regions with phantom energy ($w < -1$) or dark energy ($-1 < w < -1/3$) is balanced by the gravitational attraction coming from the regions where the field has $w > -1/3$. This is reminiscent of the Bartnik-McKinnion solution which owes its existence to the interplay with the repulsion of the Yang-Mills field against the attraction from gravity. In the present case the repulsion of the regions where the scalar field has $w < -1/3$ (and especially regions where $w < -1$) is balanced by the attraction of the regions where $w > -1/3$.

There have been studies [30] where the equation of state of some fluid or field, as given by w, varies with time. In these studies there is some point in time where scalar field makes a transition and begins to violate the WEC and becomes a phantom field. In the present work our equation of state, as given by w, is spatially varying and it is this feature which gives rise to the static, spherically symmetric solutions. A similar result can occur even for a regular, real, scalar coupled to gravity. In [31] such a system was studied and it was found that there are regions where the weak energy condition is violated. This paper also found non-singular spherically symmetric solutions.

3. Stability analysis

We now study the dynamical stability of the above solutions against linear perturbations. We perturb the solutions of the system (2.6) by expanding the metric functions and scalar field function to first order as follows [9, 28, 32]

$$\begin{aligned}
\nu(t,x) &= \nu_0(x) + \nu_1(x)\cos(\omega t), \\
\lambda(t,x) &= \lambda_0(x) + \lambda_1(x)\cos(\omega t), \\
\varphi(t,x) &= \varphi_0(x) + \varphi_1(x)\cos(\omega t)/x.
\end{aligned}$$

The index 0 indicates the static background solutions of equations (2.10)-(2.12). (Expressions for $\nu_0(x)$, $\lambda_0(x)$ can be obtained from (2.8).) Then the first-order perturbation

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equations following from (2.6) and (2.7) are

$$\lambda_1 = -\frac{1}{2}\varphi_0'\varphi_1, \tag{3.1}$$

$$\nu_1' = \frac{1}{2}\varphi_0''\varphi_1 - \frac{1}{2}\varphi_0'\varphi_1' + \frac{1}{x}\varphi_0'\varphi_1, \tag{3.2}$$

and the equation for φ_1 is

$$\varphi_1'' + \frac{1}{2}\left(\nu_0' - \lambda_0'\right)\varphi_1' - V_0(x)\varphi_1 + \omega^2 e^{\lambda_0-\nu_0}\varphi_1 = 0 \tag{3.3}$$

with the potential

$$V_0(x) = \frac{1}{2x}\left(\nu_0' - \lambda_0'\right) - \frac{1}{2}\varphi_0''\varphi_0' x - \frac{1}{2}\varphi_0'^2 - e^{\lambda_0}\left(1 - 3\Lambda\varphi_0^2\right). \tag{3.4}$$

Introducing the new independent variable ρ

$$\frac{d\rho}{dx} = e^{(\lambda_0-\nu_0)/2}$$

we can rewrite equation (3.3) in a Schrödinger-like form

$$-\frac{d^2\varphi_1}{d\rho^2} + V[x(\rho)]\varphi_1 = \omega^2\varphi_1,$$

where $V[x(\rho)] = e^{\nu_0-\lambda_0}V_0(x)$. If there is a negative eigenvalue ω^2 then the solution will be unstable since then $\varphi_1 \sim e^{i\omega t}$ will grow exponentially. To this end we examine the asymptotic behavior of the potential $V[x(\rho)]$. Using the asymptotic expressions for the metric and scalar field ansatz functions from the previous section one finds

$$V[x(\rho)] \to -e^{\nu_0} = -1 \quad \text{as} \quad x \to \infty.$$

On the other hand, as $x \to 0$ we can expand the solutions into series in the following form

$$\begin{aligned}
\varphi_0 &\sim \varphi(0) + \frac{1}{6}\varphi(0)\left[1 - \Lambda\varphi(0)^2\right]x^2, \\
\tilde{M}_0 &\sim -\frac{1}{12}\left[\varphi(0)^2 - \frac{\Lambda}{2}\varphi(0)^4\right]x^3, \\
C_0 &\sim C(0)\exp\left\{-\left[\frac{1}{3\sqrt{2}}\varphi(0)\left(1 - \Lambda\varphi(0)^2\right)\right]^2\frac{x^4}{4}\right\},
\end{aligned}$$

whence the expressions for ν_0 and λ_0 can be found (see equation (2.8)). Then, using (3.4), one finds that as $x \to 0$

$$V[x(\rho)] \sim \alpha + \beta x^2,$$

where

$$\alpha = \gamma - C(0)(1 - 3\Lambda\varphi(0)^2 - \gamma), \quad \beta = C(0)\Lambda\varphi(0)^2[1 - \Lambda\varphi(0)^2] - \gamma,$$

with

$$\gamma = \frac{1}{12}\left[1 - \frac{\Lambda}{2}\varphi(0)^2\right]\varphi(0)^2.$$

Depending on Λ and $\varphi(0)$ the coefficients α and β can take values such that near x=0 one has a positive definite potential well or not. However since $V[x(\rho)] \to -1$ as $x \to \infty$ one always has some negative ω^2 coming from continuum states. Because of these negative eigenvalues the solution is unstable.

This result is not surprising. It is well-known [33] that in 4D there are no stable soliton solutions to the Klein-Gordon equation with the "Mexican hat" potential (or any other type of potential). In [9] it was found that adding gravity did not give stable solutions. Here we have added the additional ingredient of a phantom field but as the above stability analysis shows this also does not lead to stable solutions. Note, that in [28] the stability analysis for the normal scalar field done in [9] was re-visited and it was claimed that there were some regions where ω^2 was positive definite. We do not exclude this possibility for the present phantom field case since we did not do a complete scan of the entire parameter space.

4. Conclusions

By studying the system of 4D gravity plus a phantom scalar field (a real scalar field with a negative sign in front of the kinetic energy term in the Lagrangian density) we have found finite energy, regular solutions which have trivial topology. The solutions share some common features with previous solutions to systems of 4D gravity plus some field(s). Like the Bartnik-McKinnion solutions our solutions have finite energy, are regular everywhere, and have no horizons. The physical mechanism behind the present solutions is similar to that of the Bartnik-McKinnion solutions: the existence of the Bartnik-McKinnion solutions comes from the interplay of the repulsion of the Yang-Mills fields against the attraction of gravity. The present solutions arise from the balancing of regions where the scalar field a phantom/dark equation of state (and is therefore gravitationally repulsive), against regions where the scalar field has the equation of state for ordinary matter-energy (and is therefore gravitationally attractive).

The present solutions had some different features compared to the closely related ghost field, kink solutions of [6] (for these solutions there was a negative sign in front of *both* the kinetic and potential terms in the scalar field Lagrangian density). These kink solutions had a non-trivial wormhole topology which resulted in their stability. In contrast the present solutions had a trivial topology, an everywhere regular metric and no horizons. However the linear stability analysis of the present solutions showed that they are unstable, as are the solutions of a regular scalar field plus 4D gravity and the higher Bartnik-McKinnion solutions (those with $k > 2$). One possibility toward finding stable phantom field solutions might be to alter the form of the potential $V(\varphi)$. Note that asymptotically $\varphi \to 0$ which is an *unstable* equilibrium point for a normal scalar field. This behavior is due to the reversed character of a scalar field with a negative sign in front of kinetic energy term — it should go toward maxima of the potential. Unfortunately for the "Mexican hat" potential $\varphi = 0$ is only a local maximum; the global maxima is at $\varphi = \pm\infty$. This suggests trying some potential which has a global maxima at some finite φ and with $V(\varphi)$ finite at this point.

The mass of our solutions turned out to be negative. For small Λ the magnitude of the mass was large and decreased to zero from below as $\Lambda \to \infty$ (see figure 3 and table I).

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Given the unusual nature of the scalar field this could have been expected, and is similar to negative mass black hole solutions [26]. Despite the instability and negative mass features of the present solutions there is nevertheless a physical motivation for studying this system since experimental evidence [13] favors the existence of something like phantom energy.

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